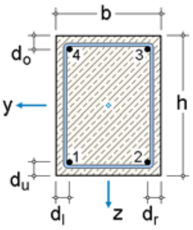
POS. 29: RECTANGLE (REINFORCED CONCRETE 2-ACHS.)

bending and shear design incl. verif. of serviceability (EC 2 (1.11), NA: Deutschland)

biaxial bending with/abs. axial force (4H-BETON version: 11/2007-4I)



rectangular section
b = 40.0 cm   h = 40.0 cm
edge distances of longit. reinf.
d_o = 6.0 cm d_u = 6.0 cm
d_l = 6.0 cm d_r = 6.0 cm
material
C25/30
BSt 500 (A)
$\gamma_s = 1.15$, $\gamma_c = 1.50$
exposure class X0

min./max. reinforcement
min A_s (9.2.1.1, 9.5.2), max ρ_0 = 8.00 %
reinforcement groups

Nr	rank	min A_s	max A_s
	-	cm^2	cm^2
1	1	0.00	100.00
2	1	0.00	100.00
3	1	0.00	100.00
4	1	0.00	100.00

min A_s: initial reinforcement per group
max A_s: highest reinforcement amount per group

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)
with $f_{cd} = \alpha_c f_{ck} / \gamma_c$ = 14.2 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $f_{yd} = f_{yk} / \gamma_s$ = 434.8 MN/m2
and $f_{td} = f_{tk} / \gamma_s$ = 456.5 MN/m2 !
verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)
with $f_c = f_{cm}$ = 33.0 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with $f_y = f_{yk}$, f_t = 525.0 MN/m2 and ε_{uk} = 25‰ !

design calculation values and minimum reinforcement areas (EC 2, 6.1)

	γ	N_{Ed}	M_{yEd}	M_{zEd}	ε_{c2u}	ε_{s2u}	ε_{s1u}	ε_{c1u}	α_{ku}	d	z	x
	-	kN	kNm	kNm	‰	‰	‰	‰	°	cm	cm	cm
1	---	-1200.0	150.00	75.00	-3.50	-2.61	1.54	2.43	120.27	46.5	29.4	32.3
			27.36	0.00	-2.68	2.21	25.00	29.88	90.00	34.0	19.6	3.3

ε_{c2u}: concr. strain in state of failure (fibre 2), ε_{s1u}: reinforcement strain in state of failure (fibre 1),
α_{ku}: dir. angle of cross section principal strain, d: static height, z: lever arm of internal forces, x: height of concr. compr. zone

	A_{sb1}	A_{sb2}	A_{sb3}	A_{sb4}	note
	cm^2	cm^2	cm^2	cm^2	
1	**4.24**	**4.24**	**4.24**	**4.24**	
	0.74	0.74	0.74	0.74	8)

8) minimum reinforcement acc. to 9.2.1.1

⇒ **longitudinal reinforcement: min A_s = 4.2/4.2/4.2/4.2 cm^2**

shear design calculation (EC 2, 6.2 + 6.3) - separated into V_{yEd} +T_{Ed} and V_{zEd} +T_{Ed}
minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf.
z = 0.9 d (10.3.4(2), d in each direction), $c_{v,D}$ = 3.0 cm, D = compression reinf.
angle of compr. strut θ_{gew} = 0°, torsion: t_{eff} = 10.0 cm
the minimum value of V_{Rdct} is limited acc. to design code (V_{Rdct} ≥ min V_{Rdct}).

design calculation of shear force (EC 2, 6.2)

	V_{yEd}	V_{zEd}	z_y	V_{yRdct}	θ_y	V_{yRdmax}	z_z	V_{zRdct}	θ_z	V_{zRdmax}	$a_{s,bü V}$	note
	kN	kN	cm	kN	°	kN	cm	kN	°	kN	cm^2/m	
1	70.00	163.00	28.0	106.25	18.4	357.00	28.0	106.25	25.2	458.17	6.29	

z: decisive inner lever arm, V_{Rdct}: design value of shear resistance without shear reinforcement
θ: angle of compr. strut, V_{Rdmax}: design value of maximal shear resistance

design calculation of torsion (EC 2, 6.3)

	T_{Ed}	V_{yEdT+V}	θ_y	V_{zEdT+V}	θ_z	T_{Rdmax}	$a_{s,bü T}$	$A_{s,T}$	note
	kNm	kN	°	kN	°	kNm	cm^2/m	cm^2	
1	5.00	25.83	18.4	49.08	25.2	51.54	0.30	2.30	

stirrup reinforcement $a_{s,bü T}$ per leg, longit. reinf. for torsion $A_{s,T}$ uniformly distributed along the perimeter

design calculation of shear force and torsion (EC 2, 6.3(4))
1: (T_{Ed} /T_{Rdmax})+(V_{yEd} /V_{yRdmax}) = **0.05 < 1.0**
(T_{Ed} /T_{Rdmax})+(V_{zEd} /V_{zRdmax}) = **0.14 < 1.0**   ⇒ verification executed !

⇒ **shear reinforcement:** min $a_{s,\text{büV}}$ = 6.29 cm^2/m

torsion: min $a_{s,\text{büT}}$ = 0.30 cm^2/m (1-shear)

Σ (2-shear) min $a_{s,\text{bü}}$ = 6.90 cm^2/m

torsion: min $A_{s,T}$ = 2.3 cm^2 (uniformly distributed along the perimeter)

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)

cracking in bending restraint (self induced)

factor for the concrete hardening process $k_{z,t}$ = 1.00

axial force in the centre of gravity at formation of first crack N_{cr} = 0.00 kN

crack width w_k = 0.30 mm

crack forces and moments: N_r = -700.00 kN M_{yr} = 180.00 kNm M_{zr} = 65.00 kNm

reinforcement (initial state): A_s = 4.24/4.24/4.24/4.24 cm^2

Nr	d_s mm	k_c -	k -	$A_{s,min}$ cm^2	d_{sgr} mm	σ_s N/mm^2	ΔA_{sr} cm^2
1	16	0.40	0.74	1.37	33.2	166.8	2.30
2	16	0.40	0.74	1.37	16.4	287.1	2.30
3	16	0.40	0.74	1.37	16.0	0.0	2.30
4	16	0.40	0.74	1.37	16.0	0.0	2.30

concr. tens. str. (restr.) $f_{ct,eff}$ = 3.00 N/mm^2 concr. tens. strength (load) $f_{ct,eff} = f_{ctm}$ = 2.56 N/mm^2

k_c: coeff. - stress distribution, k: coeff. - concr. tens. stress, As,min: min. reinf. from restraint

d_s: selected bar diameter, d_{sgr}: existing bar diameter

σ_s: steel tensile stress, ΔA_{sr}: reinf. increase from load and restraint

⇒ **incl. anti-crack reinforcement: min A_s = 6.5/6.5/6.5/6.5 cm^2**

fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: $U_{s1} = \gamma_{F,fat}\ \gamma_{Ed,fat}\ \Delta\sigma_{s,equ} \le U_{s2} = \Delta\sigma_{Rsk}(N^*)/\gamma_{s,fat}$ = 152.17 N/mm^2

damage equivalent stress range $\Delta\sigma_{s,equ} = \sigma_{s,O} - \sigma_{s,U}$

partial safety factors $\gamma_{F,fat}$ = 1.00, $\gamma_{Ed,fat}$ = 1.00, $\gamma_{s,fat} = \gamma_s$ = 1.15

allowable stress range $\Delta\sigma_{Rsk}(N^*)$ = 175.0 N/mm^2

shear force : $\Delta\sigma_{Rskv}(N^*)$ = 107.0 N/mm^2 ⇒ $U_{s2v} = \Delta\sigma_{Rskv}(N^*)/\gamma_{s,fat}$ = 93.04 N/mm^2

for conc.: $U_{c1} = |\sigma_{cd,max,equ}|/f_{cd,fat} + 0.43\ \text{sqrt}(1 - \sigma_{cd,min,equ}/\sigma_{cd,max,equ}) \le 1.0$

design value of compression strength $f_{cd,fat}$ = 15.00 N/mm^2 at t_0 = 28 d

material safety $\gamma_{c,fat} = \gamma_c$ = 1.50

load: N_{s1} = -800.00 kN M_{ys1} = 90.00 kNm M_{zs1} = 75.00 kNm V_{ys1} = 0.00 kN V_{zs1} = 163.00 kN

N_{s2} = -500.00 kN M_{ys2} = 110.00 kNm M_{zs2} = 100.00 kNm V_{ys2} = 0.00 kN V_{zs2} = 0.00 kN

reinforcement (initial state): A_s = 6.53/6.53/6.53/6.53 cm^2 $a_{s,\text{büV}}$ = 6.29 cm^2/m

fatigue design for steel:

Nr	$\sigma_{s,O}$ N/mm^2	$\sigma_{s,U}$ N/mm^2	$\Delta\sigma_{s,equ}$ N/mm^2	U_{s1} N/mm^2	$\Delta A_{s,fat}$ cm^2
1	50.83	-9.76	60.59	60.59	0.35
2	230.73	79.01	151.72	151.72	0.35
3	29.80	-25.23	55.03	55.03	0.35
4	-114.00	-150.10	36.10	36.10	0.35

reinforcement (shear force):

$\Delta\sigma_{sv,equz}$ = 93.04 - 0.00 = 93.04 N/mm^2 = U_{s2v}

⇒ $\Delta a_{sbü,fat}$ = 34.64 cm^2/m

concrete fatigue design:

$\sigma_{cd,min,equ}$ = 19.96 N/mm^2

$\sigma_{cd,max,equ}$ = 26.47 N/mm^2

U_{c1} = 1.98 > 1.00 ⇒ verification **not** complied !

verif. of compression strut for decisive $V_{s1,2}$:

$\sigma_{cdv,min,equ}$ = 0.00 N/mm^2

$\sigma_{cdv,max,equ}$ = 3.62 N/mm^2

U_{c1v} = 0.36 < 0.50 ⇒ verification executed !

⇒ **incl. fatigue reinforcement: min A_s = 6.9/6.9/6.9/6.9 cm^2**

min $a_{s,\text{büV}}$ = 40.94 cm^2/m

limitation of steel tension and concrete compression stresses (EC 2, 7.2)

permitted tensile stress of reinf. $\sigma_s = 0.80 \cdot f_{yk}$ = 400.0 N/mm^2

permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck}$ = -15.0 N/mm^2

stress forces and moments: N_σ = -1200.00 kN $M_{y\sigma}$ = 150.00 kNm $M_{z\sigma}$ = 75.00 kNm

reinforcement (initial state): A_s = 6.88/6.88/6.88/6.88 cm^2

maximal reinforcement tensile stresses

Nr	σ_{0s} N/mm^2	σ_s N/mm^2	$\Delta A_{s\sigma}$ cm^2
1	10.0	0.0	32.52
2	104.3	28.8	32.52
3	-70.9	0.0	32.52
4	-165.1	0.0	32.52

σ_{0s}: initial state, σ_s: end state
$\Delta A_{s\sigma}$: reinforcement increase from steel and concrete design

minimal concrete compression stress
initial state:
$\sigma_{0c} = -25.8$ N/mm^2
end state:
$\sigma_c = -15.0$ N/mm^2 > -15.0

⇒ **incl. stress reinforcement: min A_s = 39.4/39.4/39.4/39.4 cm^2** (max ρ_0 !)

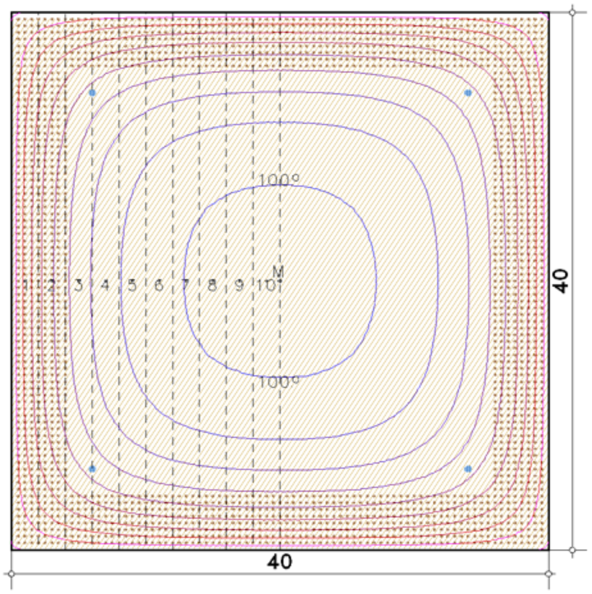
fire protection acc. to EC2, Teil 1-2 (10.06)

mod. zone method (10 zones)
column flame application from left, right, top, bottom, fire duration 90 min
convectiv coeff. of thermal transfer $\alpha = 25.0$ W/m²K, emissivity coeff. for concrete surface $\varepsilon = 0.70$
normal dens. concr. with silicious aggr., moisture content 1.5%, upper limit of thermal conduct.
hot rolled reinforcing steel, density (reinforced concrete) $\rho_c = 2300$ kg/m^3
assumption for the design calculation: concrete temperature of the coldest cross-section point (point M)
assumption for the design calculation: no inner stresses to be taken into account
assumption for the design calculation: stress-strain relation form acc. to EC 2 (fire case)

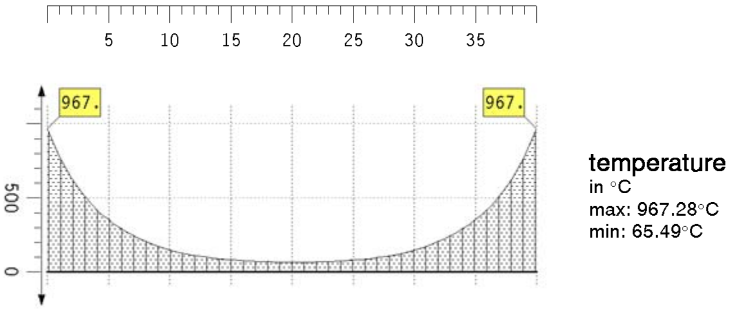
simplified method for transient heat transport

finite volume method with explicit time integration taking into account non-linear material and boundary conditions

temperature profile (90 min, rotated 0°):
dx = 0.50 cm dy = 0.50 cm (6561 cell nodes), min dt = 0.055 min



horizontal section through point M:



temperatures for 10 zones with related reduction factors:
$\Theta_1 = 788.4$°C, $k_{c1} = 0.167$ $\Theta_2 = 522.2$°C, $k_{c2} = 0.567$ $\Theta_3 = 357.4$°C, $k_{c3} = 0.793$
$\Theta_4 = 249.3$°C, $k_{c4} = 0.901$ $\Theta_5 = 175.7$°C, $k_{c5} = 0.962$ $\Theta_6 = 127.2$°C, $k_{c6} = 0.986$
$\Theta_7 = 98.8$°C, $k_{c7} = 1.000$ $\Theta_8 = 81.8$°C, $k_{c8} = 1.000$ $\Theta_9 = 71.4$°C, $k_{c9} = 1.000$
$\Theta_{10} = 66.4$°C, $k_{c10} = 1.000$
mean reduction factor (related temperature): $k_{cm} = 0.838$ ($\Theta_{cm} = 312.4$°C)
temperature in point M with associated reduction factor: $\Theta_{cM} = 65.5$°C, $k_{cM} = 1.000$
static ineffective concrete boundary zone: $a_{zl} = 4.21$ cm $a_{zr} = 4.21$ cm $a_{zo} = 4.21$ cm $a_{zu} = 4.21$ cm
concrete temperature (design calculation) with associated reduction factor: $\Theta_c = 65.5$°C, $k_c = 1.000$
reinforcement temperatures: $\Theta_{s1} = 466.8$°C $\Theta_{s2} = 466.8$°C $\Theta_{s3} = 466.8$°C $\Theta_{s4} = 466.8$°C
associated reduction factors: $k_{sy1} = 0.853$ $k_{sy2} = 0.853$ $k_{sy3} = 0.853$ $k_{sy4} = 0.853$
$k_{sp1} = 0.380$ $k_{sp2} = 0.380$ $k_{sp3} = 0.380$ $k_{sp4} = 0.380$
$k_{Es1} = 0.633$ $k_{Es2} = 0.633$ $k_{Es3} = 0.633$ $k_{Es4} = 0.633$

fire protection for $\gamma_c = \gamma_s = 1$ (parameters of stress-strain relation acc. to 3.2)
reduced cross-section: b = 31.58 cm h = 31.58 cm
design calculation values: $N_{Ed,fi} = -1200.00$ kN $M_{yEd,fi} = 150.00$ kNm $M_{zEd,fi} = 75.00$ kNm
material properties:
concrete $\Theta = 65$°C: $f_{c,\Theta} = 25.0$ N/mm^2 ($E_{c,\Theta} = 31475.8$ N/mm^2)
$\varepsilon_{c1,\Theta} = \varepsilon_{cu1,\Theta} = -3.35$‰ $\varepsilon_{cV,\Theta} = 0.00$‰
reinf.gr.1 $\Theta = 467$°C: $f_{sp,\Theta} = 190.0$ N/mm^2 $f_{sy,\Theta} = 426.5$ N/mm^2 $E_{s,\Theta} = 126640.3$ N/mm^2
$\varepsilon_{sp,\Theta} = 1.50$‰ $\varepsilon_{sy,\Theta} = 20.00$‰ $\varepsilon_{st,\Theta} = \varepsilon_{su,\Theta} = 50.00$‰ $\varepsilon_{sV,\Theta} = 0.00$‰
reinf.gr.2 $\Theta = 467$°C: $f_{sp,\Theta} = 190.0$ N/mm^2 $f_{sy,\Theta} = 426.5$ N/mm^2 $E_{s,\Theta} = 126640.3$ N/mm^2

$\varepsilon_{sp,\Theta}$ = 1.50‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta} = \varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

reinf.gr.3 Θ = 467°C: $f_{sp,\Theta}$ = 190.0 N/mm^2 $f_{sy,\Theta}$ = 426.5 N/mm^2 $E_{s,\Theta}$ = 126640.3 N/mm^2

$\varepsilon_{sp,\Theta}$ = 1.50‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta} = \varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

reinf.gr.4 Θ = 467°C: $f_{sp,\Theta}$ = 190.0 N/mm^2 $f_{sy,\Theta}$ = 426.5 N/mm^2 $E_{s,\Theta}$ = 126640.3 N/mm^2

$\varepsilon_{sp,\Theta}$ = 1.50‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta} = \varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

⇒ fire reinforcement: min $A_{s,\Theta}$ = 9.67/9.67/9.67/9.67 cm^2

total reinforc.: total A_s = 39.4/39.4/39.4/39.4 cm^2 (max ρ_0 !)
total $a_{s,büV}$ = 40.94 cm^2/m
total $a_{s,büT}$ = 0.30 cm^2/m, $A_{s,T}$ = 2.3 cm^2
degree of utilization: U = 0.29

selected:
longitudinal, E1: 1 Ø 25 = 4.9 cm^2 < 39.4 cm^2
E2: 1 Ø 25 = 4.9 cm^2 < 39.4 cm^2
E3: 1 Ø 25 = 4.9 cm^2 < 39.4 cm^2
E4: 1 Ø 25 = 4.9 cm^2 < 39.4 cm^2
stirrups, 2-shear: Ø 8 / 30 cm = 3.35 cm^2/m < 41.54 cm^2/m

cross-section data

gross area of concrete: A_c = 16.0 dm^2 second moment of areae: I_{cys} = 21.3 dm^4, I_{czs} = 21.3 dm^4
centroid coordinates (from centre of upper edge): y_s = 0.0 cm, z_s = 20.0 cm
total area of longitudinal reinforcement: Σ(min A_s) = 157.6 cm^2 ⇒ ρ_s = 9.85% > 8.00%

material properties for design calculation

concrete	f_{ck}	α	ε_{c2}	ε_{c2u}	n_c	E_{cm}	f_{ctm}
	MN/m^2	-	‰	‰	-	MN/m^2	MN/m^2
C25/30	25.0	0.850	-2.00	-3.50	2.00	31475.8	2.565

design value of compression strength $f_{cd} = \alpha_c f_{ck} / \gamma_c$
strain at reaching the maximum strength ε_{c2}, ult. compr. strain ε_{c2u}
concr. comp. stress $\sigma_c = f_{cd} (1-(1-\varepsilon_c/\varepsilon_{c2})^n)$ for $0 \geq \varepsilon_c > \varepsilon_{c2}$ and $\sigma_c = f_{cd}$ for $\varepsilon_{c2} \geq \varepsilon_c > \varepsilon_{c2u}$
modulus of elasticity E_{cm}, mean value of axial tensil strength f_{ctm}

reinforcem.	f_{yk}	f_{tk}	ε_{su}	E_s
	MN/m^2	MN/m^2	‰	MN/m^2
BSt 500 (A)	500.0	525.0	25.00	200000.0

design yield strength $f_{yd} = f_{yk} / \gamma_s$
design tensile strength $f_{td} = f_{tk} / \gamma_s$
ult. tensile strain ε_{su}, modulus of elasticity E_s

symbols: positive result values marked with -1.0 or **** in tables
refer to incorrect resp. not computable conditions !