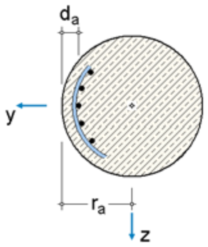
POS. 25: CIRCLE (REINFORCED CONCRETE)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)

uniaxial bending with/without axial force (4H-BETON version: 11/2007-4l)



circular cross section
r_a = 20.0 cm ⇒ $Ø_a$ = 40.0 cm
edge distances of longit. reinf.
d_a = 5.5 cm
material
C25/30
BSt 500 (A)
γ_s = 1.15, γ_c = 1.50
exposure class X0

detailing of reinforcement
circumferential reinforcement
min./max. reinforcement
min A_s (9.5.2), max ρ_0 = 8.00%
initial reinforcement
A_{s0a} = 0.00 cm^2
$a_{s0bü}$ = 0.00 cm^2/m

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)
with fcd = αc fck / γc = 14.2 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with fyd = fyk / γs = 434.8 MN/m2
and ftd = ftk / γs = 456.5 MN/m2 !
verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)
with fc = fcm = 33.0 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with fy = fyk, ft = 525.0 MN/m2 and εuk = 25‰ !

design calculation values and minimum reinforcement areas (EC 2, 6.1)

	γ	N_{Ed}	M_{Ed}	ε_{c2u}	ε_{s2u}	ε_{s1u}	ε_{c1u}	ξ	ζ	d	b_w	A_{si}	A_{sa}	note
	-	kN	kNm	‰	‰	‰	‰	-	-	cm	cm	cm^2	cm^2	
1	---	-1500.0	126.00	-3.50	-2.82	0.77	1.45	0.85	0.60	33.2	37.8	----	**24.82**	

ε_{c2u} = -3.50‰: concr. strain in state of failure (fibre 2), ε_{s1u} = 25.00‰: reinforcement strain in state of failure (fibre 1)
x = ξ d: height of conc. compr. zone, z = ζ d: lever arm of internal forces, d: effective depth, b_w: effective cross section width
b_w acc. to NABau (01/05): smallest cross section width perpendicular to inner lever arm z in height of internal forces

⇒ **longitudinal reinforcement: min A_{sa} = 24.8 cm^2**

shear design calculation (EC 2, 6.2 + 6.3)
minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf.
z = 0.9 d (6.2.3(1)), $c_{v,D}$ = 3.0 cm, D = compression reinf.
angle of reinforcement α = 90.0°, angle of compr. strut θ_{gew} = 0°, efficiency factor α_k = 0.90
the minimum value of V_{Rdct} is limited acc. to design code (V_{Rdct} ≥ min V_{Rdct}).

design calculation of shear force (EC 2, 6.2)

	V_{Ed}	ρ_l	z	V_{Rdct}	θ	cot θ	V_{Rdmax}	AB	a_l	$a_{s,büv}$	note
	kN	%	cm	kN	°		kN		cm	cm^2/m	
1	250.00	1.98	27.2	124.51	27.8	1.90	451.13	2	25.8	12.36	

ρ_l: ratio of longit. reinf. related to static height, z: decisive inner lever arm
V_{Rdct}: design value of shear resistance without shear reinforcement, θ: angle of compr. strut,
V_{Rdmax}: design value of maximal shear resistance, a_l: shift rule
AB: range of utilization see NA-DE
longit. reinf. of bar-shaped compression members with e_d/h < 0.30 should be held transverse reinf. acc. to 9.5.3(6) !

⇒ **shear reinforcement: min $a_{s,bü}$ = 12.36 cm^2/m**

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)
cracking due to centr. restraint (intrinsically imposed)
factor for progress of hardening $k_{z,t}$ = 1.00
crack width w_k = 0.25 mm
sel. diameter d_{sa} = 10 mm
crack forces and moments:
N_r = -1700.00 kN M_r = 220.00 kNm
initial state: A_{sa} = 24.82 cm^2

minimum reinforcement:
coeff. - stress distribution k_c = 1.00
coeff. - self-equil. stresses k = 0.77
concr. tens. str. (restr.) $f_{ct,eff}$ = 2.56 N/mm^2
tension zone A_{cta} = 6.3 dm^2
($A_{sta,min}$ = 8.9 cm^2)
crack control:
concr. tens. strength (load) $f_{ct,eff}$ = f_{ctm} = 2.56 N/m
σ_{sa} = 230.2 N/mm^2
($A_{sta,ste}$ = 24.8 cm^2 (⇒ d_{sa} = 35.3 mm > 10))
verification not required !

⇒ **no additional anti-crack reinforcement !**

fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: $U_{s1} = \gamma_{F,fat}\ \gamma_{Ed,fat}\ \Delta\sigma_{s,equ} \le U_{s2} = \Delta\sigma_{Rsk}(N^*)/\gamma_{s,fat}$ = 152.17 N/mm²

damage equivalent stress range $\Delta\sigma_{s,equ} = \sigma_{s,0} - \sigma_{s,U}$

partial safety factors $\gamma_{F,fat}$ = 1.00, $\gamma_{Ed,fat}$ = 1.00, $\gamma_{s,fat} = \gamma_s$ = 1.15

allowable stress range $\Delta\sigma_{Rsk}(N^*)$ = 175.0 N/mm²

shear force : $\Delta\sigma_{Rskv}(N^*)$ = 107.0 N/mm² ⇒ $U_{s2v} = \Delta\sigma_{Rskv}(N^*)/\gamma_{s,fat}$ = 93.04 N/mm²

for conc.: $U_{c1} = |\sigma_{cd,max,equ}|/f_{cd,fat} + 0.43\ \mathrm{sqrt}(1 - \sigma_{cd,min,equ}/\sigma_{cd,max,equ}) \le 1.0$

design value of compression strength $f_{cd,fat}$ = 15.00 N/mm² at t_0 = 28 d

material safety $\gamma_{c,fat} = \gamma_c$ = 1.50

load: N_{s1} = -1200.00 kN M_{s1} = 86.00 kNm V_{s1} = 250.00 kN

N_{s2} = -1450.00 kN M_{s2} = 105.00 kNm V_{s2} = 145.00 kN

reinforcement (initial state): A_{sa} = 24.82 cm² $a_{s,büV}$ = 12.36 cm²/m

fatigue design for steel:

initial state:

$\Delta\sigma_{s0a,equ}$ = -126.13 - -158.90 = 32.77 N/mm²

= end state

reinforcement (shear force):

$\Delta\sigma_{sv,equ}$ = 203.15 - 110.10 = 93.04 N/mm² = U_{s2v}

⇒ $\Delta a_{sbü,fat}$ = 35.00 cm²/m

concrete fatigue design:

$\sigma_{cd,min,equ}$ = 19.63 N/mm²

$\sigma_{cd,max,equ}$ = 23.36 N/mm²

U_{c1} = 1.73 > 1.00 ⇒ verification **not** complied !

verification of compression strut:

$\sigma_{cdv,min,equ}$ = 5.18 N/mm²

$\sigma_{cdv,max,equ}$ = 8.28 N/mm²

U_{c1v} = 0.74 > 0.71 ⇒ verification **not** complied !

⇒ **incl. fatigue reinforcement: min A_{sa} = 24.8 cm²**

min $a_{s,büV}$ = 47.37 cm²/m

⇒ **fatigue design for concrete not complied !**

limitation of steel tension and concrete compression stresses (EC 2, 7.2)

permitted tensile stress of reinf. $\sigma_s = 0.80 \cdot f_{yk}$ = 400.0 N/mm²

permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck}$ = -15.0 N/mm²

stress forces and moments: N_σ = -1500.00 kN, M_σ = 126.00 kNm

reinforcement (initial state): A_{sa} = 24.82 cm²

maximal reinforcement tensile stresses

initial state:

σ_{0sa} = 44.1 N/mm²

end state:

σ_{sa} = 11.2 N/mm² < 400.0 N/mm²

minimal concrete compression stress

initial state:

σ_{0c} = -26.5 N/mm²

end state:

σ_c = -14.8 N/mm² > -15.0 N/mm²

⇒ $\Delta A_{s\sigma a}$ = 170.8 cm²

⇒ **incl. stress reinforcement: min A_{sa} = 195.6 cm²** (max ρ_0 !)

fire protection acc. to EC2, Teil 1-2 (10.06)

mod. zone method (10 zones)

column flame application from all sides, fire duration 90 min

convectiv coeff. of thermal transfer α = 25.0 W/m²K, emissivity coeff. for concrete surface ε = 0.70

normal dens. concr. with silicious aggr., moisture content 1.5%, upper limit of thermal conduct.

hot rolled reinforcing steel, density (reinforced concrete) ρ_c = 2300 kg/m³

assumption for the design calculation: concrete temperature of the coldest cross-section point (point M)

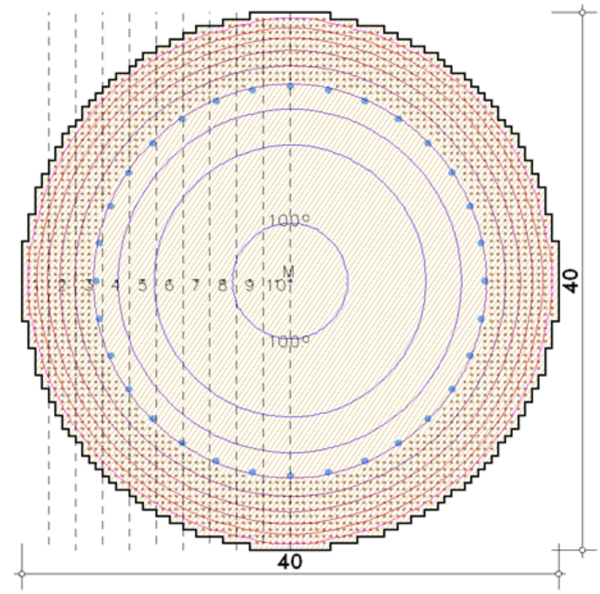
assumption for the design calculation: no inner stresses to be taken into account

assumption for the design calculation: stress-strain relation form acc. to EC 2 (fire case)

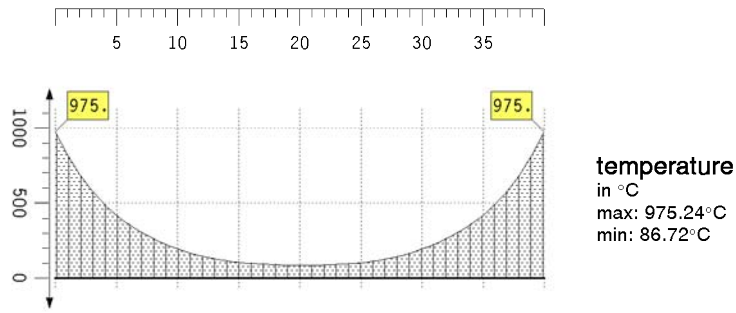
simplified method for transient heat transport

finite volume method with explicit time integration taking into account non-linear material and boundary conditions

temperature profile (90 min, rotated 0°):
dx = 0.50 cm dy = 0.50 cm (6561 cell nodes), min dt = 0.055 min



horizontal section through point M:



temperatures for 10 zones with related reduction factors:

Θ_1 = 826.1°C, k_{c1} = 0.132 Θ_2 = 588.0°C, k_{c2} = 0.468 Θ_3 = 425.0°C, k_{c3} = 0.712
Θ_4 = 311.2°C, k_{c4} = 0.839 Θ_5 = 229.3°C, k_{c5} = 0.921 Θ_6 = 169.8°C, k_{c6} = 0.965
Θ_7 = 129.2°C, k_{c7} = 0.985 Θ_8 = 105.6°C, k_{c8} = 0.997 Θ_9 = 93.6°C, k_{c9} = 1.000
Θ_{10} = 87.8°C, k_{c10} = 1.000

mean reduction factor (related temperature): k_{cm} = 0.802 (Θ_{cm} = 348.1°C)
temperature in point M with related reduction factors: Θ_{cM} = 86.7°C, k_{cM} = 1.000
static ineffective concrete boundary zone: a_z = 5.10 cm
concrete temperature (design calculation) with associated reduction factor: Θ_c = 86.7°C, k_c = 1.000
reinforcement temperatures: Θ_{sa} = 395.5°C
associated reduction factors: $k_{sy,a}$ = 1.000 $k_{sp,a}$ = 0.429 $k_{Es,a}$ = 0.705

fire protection for $\gamma_c=\gamma_s=1$ (parameters of stress-strain relation acc. to 3.2)
reduced cross-section radius: r = 14.90 cm
design calculation values: $N_{Ed,fi}$ = -1500.00 kN $M_{yEd,fi}$ = 126.00 kNm
material properties:
concr. Θ_c = 87°C: $f_{c,\Theta}$ = 25.0 N/mm² ($E_{c,\Theta}$ = 31475.8 N/mm²)
$\varepsilon_{c1,\Theta}$ = $\varepsilon_{cu1,\Theta}$ = -3.75‰ $\varepsilon_{cV,\Theta}$ = 0.00‰
reinf. Θ_s = 395°C: $f_{sp,\Theta}$ = 214.3 N/mm² $f_{sy,\Theta}$ = 500.0 N/mm² $E_{s,\Theta}$ = 140906.1 N/mm²
$\varepsilon_{sp,\Theta}$ = 1.52‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta}$ = $\varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

⇒ fire reinforcement: min A_{sa} = 46.12 cm²

total reinforc.: total A_{sa} = 195.6 cm² (max ρ_0 !)
total $a_{s,büV}$ = 47.37 cm²/m
fatigue design for concrete not complied !

selected:

longitudinal, outer face: 8 Ø 20 = 25.1 cm² < 195.6 cm²
stirrups, outer face: Ø 10 / 30 cm = 5.24 cm²/m < 47.37 cm²/m

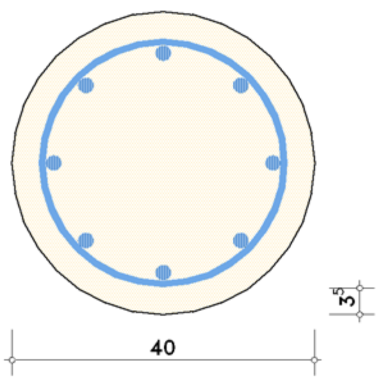
anchorage lengths outer face ($A_{sb,min}$ = 24.82 cm² $A_{s,exis}$ = 25.13 cm²):
l_b: basic size of anchorage length, $l_{b,min}$: minimum value of anchorage length, $l_{b,net}$: anchorage length
curt. of longit. tension reinf.: anch. l. at $l_{b,dir}$: direct end support, $l_{b,ind}$: indirect end support, $l_{b,Zwi}$: intermediate support

with hooks:	l_b = 80.7 cm,	$l_{b,min}$ = 20.0 cm,	$l_{b,net}$ = 55.8 cm
	$l_{b,dir}$ = 37.2 cm,	$l_{b,ind}$ = 55.8 cm,	$l_{b,Zwi}$ = 12.0 cm
without:	l_b = 80.7 cm,	$l_{b,min}$ = 24.2 cm,	$l_{b,net}$ = 79.7 cm
	$l_{b,dir}$ = 53.1 cm,	$l_{b,ind}$ = 79.7 cm,	$l_{b,Zwi}$ = 12.0 cm

reinforcement drawing:

scale 1 : 10

cv = 3.5 cm



cross-section data

gross area of concrete: A_c = 12.6 dm², second moment of area: I_{cs} = 12.6 dm⁴

distance of centre of gravity from upper edge: z_s = 20.0 cm

total area of longitudinal reinforcement: $\Sigma(\min A_s)$ = 195.6 cm² ⇒ ρ_s = 15.56% > 8.00%

material properties for design calculation

concrete	f_{ck}	α	ε_{c2}	ε_{c2u}	n_c	E_{cm}	f_{ctm}
	MN/m²	-	‰	‰	-	MN/m²	MN/m²
C25/30	25.0	0.850	-2.00	-3.50	2.00	31475.8	2.565

design value of compression strength $f_{cd} = \alpha_c f_{ck} / \gamma_c$

strain at reaching the maximum strength ε_{c2}, ult. compr. strain ε_{c2u}

concr. comp. stress $\sigma_c = f_{cd} (1-(1-\varepsilon_c/\varepsilon_{c2})^n)$ for $0 \geq \varepsilon_c > \varepsilon_{c2}$ and $\sigma_c = f_{cd}$ for $\varepsilon_{c2} \geq \varepsilon_c > \varepsilon_{c2u}$

modulus of elasticity E_{cm}, mean value of axial tensil strength f_{ctm}

reinforcem.	f_{yk}	f_{tk}	ε_{su}	E_s
	MN/m²	MN/m²	‰	MN/m²
BSt 500 (A)	500.0	525.0	25.00	200000.0

design yield strength $f_{yd} = f_{yk} / \gamma_s$

design tensile strength $f_{td} = f_{tk} / \gamma_s$

ult. tensile strain ε_{su}, modulus of elasticity E_s

symbols: positive result values marked with -1.0 or **** in tables refer to incorrect resp. not computable conditions !