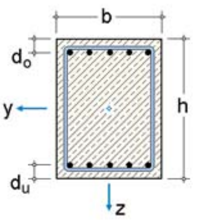
POS. 17: RECTANGLE (REINFORCED CONCRETE 1-ACHS.)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)

uniaxial bending with/without axial force (4H-BETON version: 11/2007-4I)



rectangular section
b = 20.0 cm, h = 40.0 cm
edge distances of longit. reinf.
d_o = 3.8 cm, d_u = 5.3 cm
material
C25/30
BSt 500 (A)
γ_s = 1.15, γ_c = 1.50
exposure class X0

detailing of reinforcement
preferably in tension face (ε_{s1u} = 25.00‰)
min./max. reinforcement
min A_s (9.2.1.1, 9.5.2), max ρ_0 = 8.00%
initial reinforcement
A_{s0o} = 0.00 cm^2, A_{s0u} = 0.00 cm^2
$a_{s0bü}$ = 0.00 cm^2/m

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3) with fcd = αc fck / γc = 14.2 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with fyd = fyk / γs = 434.8 MN/m2 and ftd = ftk / γs = 456.5 MN/m2 !
verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2) with fc = fcm = 33.0 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with fy = fyk, ft = 525.0 MN/m2 and εuk = 25‰ !
in deformation analysis the influence of creep and shrinkage is taken into account by modification of the concrete stress-strain relation with the creep coefficient φeff = 2.897 and the shrinkage strain εcs = -0.523‰ (φeff, εcs calculated with: cement grade 32.5 R, relative humidity RH = 50%, perimeter in contact with the atmoshere U = 120.0 cm, gross area Ac = 800.0 cm2, loading t0k = 28 d, permanent loading factor feff = M1perm/M1Ed = 1.00)).

design calculation values and minimum reinforcement areas (EC 2, 6.1)

	γ	N_{Ed}	M_{Ed}	ε_{c2u}	ε_{s2u}	ε_{s1u}	ε_{c1u}	ξ	ζ	d	A_{so}	A_{su}	note
	-	kN	kNm	‰	‰	‰	‰	-	-	cm	cm^2	cm^2	
1	---	-20.00	82.50	-3.50	-2.45	6.10	7.56	0.36	0.85	34.7	----	**6.16**	
			13.68	-1.77	1.17	25.00	29.09	----	----	----	----	0.81	9)

ε_{c2u} = -3.50‰: concr. strain in state of failure (fibre 2), ε_{s1u} = 25.00‰: reinforcement strain in state of failure (fibre 1)
x = ξ d: height of conc. compr. zone, z = ζ d: lever arm of internal forces, d = h - d_1: effective depth

9) minimum reinforcement acc. to 9.2.1.1

⇒ **longitudinal reinforcement: min A_{so} = 0.0 cm^2 min A_{su} = 6.2 cm^2**

shear design calculation (EC 2, 6.2 + 6.3)
minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf.
z = 0.9 d (6.2.3(1)), $c_{v,D}$ = 3.0 cm, D = compression reinf.
angle of reinforcement α = 90.0°, angle of compr. strut θ_{gew} = 0°
the minimum value of V_{Rdct} is limited acc. to design code (V_{Rdct} ≥ min V_{Rdct}).

design calculation of shear force (EC 2, 6.2)

	V_{Ed}	ρ_l	z	V_{Rdct}	θ	cot θ	V_{Rdmax}	AB	a_l	$a_{s,büv}$	note
	kN	%	cm	kN	°		kN		cm	cm^2/m	
1	50.00	0.89	28.7	36.40	18.4	3.00	182.96	1	43.0	1.64	minimum reinforcement

ρ_l: ratio of longit. reinf. related to static height, z: decisive inner lever arm
V_{Rdct}: design value of shear resistance without shear reinforcement, θ: angle of compr. strut,
V_{Rdmax}: design value of maximal shear resistance, a_l: shift rule
AB: range of utilization see NA-DE

⇒ **shear reinforcement: min $a_{s,bü}$ = 1.64 cm^2/m**

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)

cracking due to centr. restraint (intrinsically imposed)
factor for progress of hardening $k_{z,t}$ = 1.00
min. tensile strength required if $k_{z,t} \geq 1$
crack width w_k = 0.25 mm
sel. diameter d_{so} = 10 mm d_{su} = 20 mm
crack forces and moments:
N_r = 40.00 kN M_r = 34.00 kNm
initial state: A_{so} = 0.00 cm^2 A_{su} = 6.16 cm^2

minimum reinforcement:
coeff. - stress distribution k_c = 1.00
coeff. - self-equil. stresses k = 0.80
concr. tens. str. (restr.) $f_{ct,eff}$ = 3.00 N/mm^2
tension zones A_{cto} = 4.0 dm^2 A_{ctu} = 4.0 dm^2
($A_{sto,min}$ = 2.3 cm^2 $A_{stu,min}$ = 4.1 cm^2)

crack control:
concr. tens. strength (load) $f_{ct,eff}$ = f_{ctm} = 2.56 N/m
σ_{so} = 0.0 N/mm^2 σ_{su} = 219.1 N/mm^2
($A_{sto,ste}$ = 0.0 cm^2 (d_{so} = 10 mm)
$A_{stu,ste}$ = 6.2 cm^2 ($\Rightarrow d_{su}$ = 25.3 mm > 20))

additional reinforcement:
max A_{sto} = 2.3 cm^2 $\Rightarrow$ ΔA_{sto} = 2.3 cm^2

$\Rightarrow$ incl. anti-crack reinforcement: min A_{so} = 2.3 cm^2 min A_{su} = 6.2 cm^2

fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: $U_{s1} = \gamma_{F,fat}\ \gamma_{Ed,fat}\ \Delta\sigma_{s,equ} \leq U_{s2} = \Delta\sigma_{Rsk}(N^*)/\gamma_{s,fat}$ = 152.17 N/mm^2
damage equivalent stress range $\Delta\sigma_{s,equ} = \sigma_{s,O} - \sigma_{s,U}$
partial safety factors $\gamma_{F,fat}$ = 1.00, $\gamma_{Ed,fat}$ = 1.00, $\gamma_{s,fat} = \gamma_s$ = 1.15
allowable stress range $\Delta\sigma_{Rsk}(N^*)$ = 175.0 N/mm^2
shear force : $\Delta\sigma_{Rskv}(N^*)$ = 107.0 N/mm^2 $\Rightarrow U_{s2v} = \Delta\sigma_{Rskv}(N^*)/\gamma_{s,fat}$ = 93.04 N/mm^2
for conc.: $U_{c1} = |\sigma_{cd,max,equ}|/f_{cd,fat} + 0.43\ \mathrm{sqrt}(1 - \sigma_{cd,min,equ}/\sigma_{cd,max,equ}) \leq 1.0$
design value of compression strength $f_{cd,fat}$ = 15.00 N/mm^2 at t_0 = 28 d
material safety $\gamma_{c,fat} = \gamma_c$ = 1.50
load: N_{s1} = -10.00 kN M_{s1} = 62.50 kNm V_{s1} = 50.00 kN
N_{s2} = 12.00 kN M_{s2} = 75.00 kNm V_{s2} = 30.00 kN
reinforcement (initial state): A_{so} = 2.32 cm^2 A_{su} = 6.16 cm^2 $a_{s,büV}$ = 1.64 cm^2/m

fatigue design for steel:
initial state:
$\Delta\sigma_{s0o,equ}$ = -218.69 - -255.54 = 36.85 N/mm^2
$\Delta\sigma_{s0u,equ}$ = 418.35 - 333.72 = 84.63 N/mm^2
= end state

reinforcement (shear force):
$\Delta\sigma_{sv,equ}$ = 232.61 - 139.57 = 93.04 N/mm^2 = U_{s2v}
$\Rightarrow$ $\Delta a_{sbü,fat}$ = 2.68 cm^2/m

concrete fatigue design:
$\sigma_{cd,min,equ}$ = 9.99 N/mm^2
$\sigma_{cd,max,equ}$ = 11.60 N/mm^2
U_{c1} = 0.93 < 1.00 $\Rightarrow$ verification executed !

verification of compression strut:
$\sigma_{cdv,min,equ}$ = 1.74 N/mm^2
$\sigma_{cdv,max,equ}$ = 2.90 N/mm^2
U_{c1v} = 0.26 < 0.57 $\Rightarrow$ verification executed !

$\Rightarrow$ incl. fatigue reinforcement: min A_{so} = 2.3 cm^2 min A_{su} = 6.2 cm^2
min $a_{s,büV}$ = 4.32 cm^2/m

limitation of steel tension and concrete compression stresses (EC 2, 7.2)

permitted tensile stress of reinf. $\sigma_s = 0.80 \cdot f_{yk}$ = 400.0 N/mm^2
permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck}$ = -15.0 N/mm^2
stress forces and moments: N_σ = -20.00 kN, M_σ = 82.50 kNm
reinforcement (initial state): A_{so} = 2.32 cm^2 A_{su} = 6.16 cm^2

maximal reinforcement tensile stresses
initial state:
σ_{0so} = -295.0 N/mm^2 σ_{0su} = 436.2 N/mm^2
end state:
σ_{so} = -290.7 N/mm^2 < 400.0 N/mm^2
σ_{su} = 398.4 N/mm^2 < 400.0 N/mm^2
$\Rightarrow$ $\Delta A_{s\sigma u}$ = 0.6 cm^2

minimal concrete compression stress
initial state:
σ_{0c} = -13.0 N/mm^2
end state:
σ_c = -12.7 N/mm^2 > -15.0 N/mm^2

$\Rightarrow$ incl. stress reinforcement: min A_{so} = 2.3 cm^2 min A_{su} = 6.8 cm^2

verification of impermeability
DAfStb-Richtlinie: Wasserundurchlässige Bauwerke aus Beton

internal forces and moments: N_D = -20.00 kN, M_D = 82.50 kNm
reinforcement (initial state): A_{so} = 2.32 cm^2 A_{su} = 6.78 cm^2
minimal allowable height of compression zone zul x_D = 20.0 mm

verification of minimum height of compression zone:
for service class A and stress class 1

x_{min} = 168.4 mm > 20.0 mm ⇒ **verification executed**

⇒ **no additional reinforcement !**

fire protection acc. to EC2, Teil 1-2 (10.06)

mod. zone method (10 zones)

column flame application from top, bottom, left and right, fire duration 90 min

convectiv coeff. of thermal transfer α = 25.0 W/m²K, emissivity coeff. for concrete surface ε = 0.70

normal dens. concr. with silicious aggr., moisture content 1.5%, upper limit of thermal conduct.

hot rolled reinforcing steel, density (reinforced concrete) ρ_c = 2300 kg/m³

assumption for the design calculation: concrete temperature of the coldest cross-section point (point M)

assumption for the design calculation: no inner stresses to be taken into account

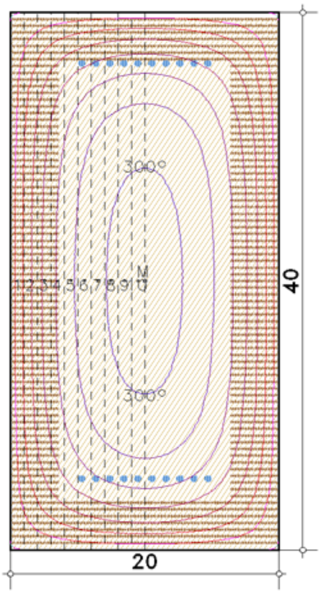
assumption for the design calculation: stress-strain relation form acc. to EC 2 (fire case)

simplified method for transient heat transport

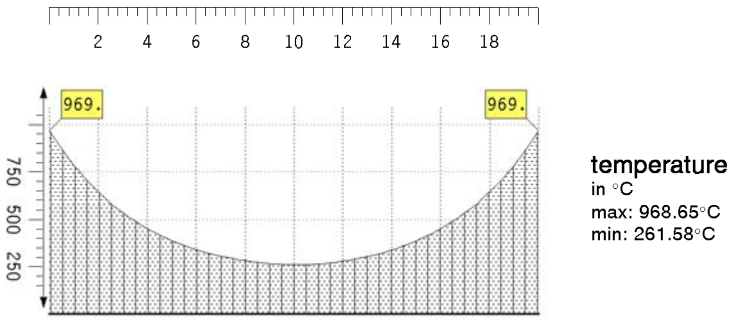
finite volume method with explicit time integration taking into account non-linear material and boundary conditions

temperature profile (90 min, rotated 0°):

dx = 0.25 cm dy = 0.50 cm (6561 cell nodes), min dt = 0.014 min



horizontal section through point M:



temperatures for 10 zones with related reduction factors:

Θ_1 = 874.6°C, k_{c1} = 0.098 Θ_2 = 712.5°C, k_{c2} = 0.281 Θ_3 = 588.0°C, k_{c3} = 0.468

Θ_4 = 492.6°C, k_{c4} = 0.611 Θ_5 = 419.3°C, k_{c5} = 0.721 Θ_6 = 363.5°C, k_{c6} = 0.787

Θ_7 = 321.7°C, k_{c7} = 0.828 Θ_8 = 291.9°C, k_{c8} = 0.858 Θ_9 = 272.7°C, k_{c9} = 0.877

Θ_{10} = 263.3°C, k_{c10} = 0.887

mean reduction factor (related temperature): k_{cm} = 0.642 (Θ_{cm} = 472.3°C)

temperature in point M with related reduction factors: Θ_{cM} = 261.6°C, k_{cM} = 0.888

static ineffective concrete boundary zone: a_{zo} = 3.52 cm a_{zu} = 3.52 cm a_{zr} = 3.52 cm

concrete temperature (design calculation) with associated reduction factor: Θ_c = 261.6°C, k_c = 0.888

reinforcement temperatures: Θ_{so} = 621.4°C Θ_{su} = 546.1°C

associated reduction factors: $k_{sy,o}$ = 0.419 $k_{sy,u}$ = 0.637

$k_{Es,o}$ = 0.271 $k_{Es,u}$ = 0.466

fire protection for $\gamma_c=\gamma_s=1$ (parameters of stress-strain relation acc. to 3.2)

reduced cross-section: b = 12.96 cm h = 32.96 cm

design calculation values: $N_{Ed,fi}$ = -20.00 kN $M_{yEd,fi}$ = 82.50 kNm

material properties:

concr. Θ_c = 262°C: $f_{c,\Theta}$ = 22.2 N/mm² ($E_{c,\Theta}$ = 24843.5 N/mm²)

$\varepsilon_{c1,\Theta}$ = $\varepsilon_{cu1,\Theta}$ = -6.42‰ $\varepsilon_{cV,\Theta}$ = 0.00‰

reinf. Θ_{su} = 546°C: $f_{sp,\Theta}$ = 138.5 N/mm² $f_{sy,\Theta}$ = 318.6 N/mm² $E_{s,\Theta}$ = 93267.1 N/mm²

$\varepsilon_{sp,\Theta}$ = 1.49‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta}$ = $\varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

reinf. Θ_{so} = 621°C: $f_{sp,\Theta}$ = 78.2 N/mm² $f_{sy,\Theta}$ = 209.3 N/mm² $E_{s,\Theta}$ = 54282.8 N/mm²

$\varepsilon_{sp,\Theta}$ = 1.44‰ $\varepsilon_{sy,\Theta}$ = 20.00‰ $\varepsilon_{st,\Theta}$ = $\varepsilon_{su,\Theta}$ = 50.00‰ $\varepsilon_{sV,\Theta}$ = 0.00‰

⇒ **fire reinforcement: min A_{so} = 0.00 cm² min A_{su} = 15.27 cm²**

total reinforc.: total A_{so} = 2.3 cm^2 A_{su} = 15.3 cm^2
total $a_{s,büV}$ = 4.32 cm^2/m

selected:

longit., top: 2 Ø 10 = 1.6 cm^2 < 2.3 cm^2
bottom: 2 Ø 20 = 6.3 cm^2 < 15.3 cm^2
torsion: 7 Ø 8 = **3.5 cm^2** > 0.0 cm^2
stirrups, 2-shear: Ø 8 / 10 cm = **10.05 cm^2/m** > 4.32 cm^2/m

anchorage lengths top ($A_{sb,min}$ = 0.00 cm^2 $A_{s,exis}$ = 1.57 cm^2):

l_b: basic size of anchorage length, $l_{b,min}$: minimum value of anchorage length, $l_{b,net}$: anchorage length
curt. of longit. tension reinf.: anch. l. at $l_{b,dir}$: direct end support, $l_{b,ind}$: indirect end support, $l_{b,Zwi}$: intermediate support

with hooks:	l_b = 57.7 cm,	$l_{b,min}$ = 12.1 cm,	$l_{b,net}$ = 12.1 cm
	$l_{b,dir}$ = 8.1 cm,	$l_{b,ind}$ = 12.1 cm,	$l_{b,Zwi}$ = 6.0 cm
without:	l_b = 57.7 cm,	$l_{b,min}$ = 17.3 cm,	$l_{b,net}$ = 17.3 cm
	$l_{b,dir}$ = 11.5 cm,	$l_{b,ind}$ = 17.3 cm,	$l_{b,Zwi}$ = 6.0 cm

anchorage lengths bottom ($A_{sb,min}$ = 6.16 cm^2 $A_{s,exis}$ = 6.28 cm^2):

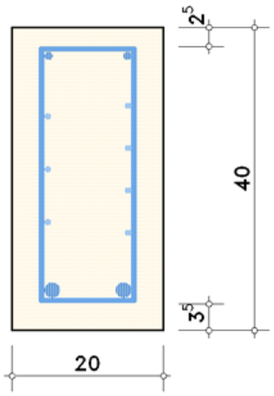
l_b: basic size of anchorage length, $l_{b,min}$: minimum value of anchorage length, $l_{b,net}$: anchorage length
curt. of longit. tension reinf.: anch. l. at $l_{b,dir}$: direct end support, $l_{b,ind}$: indirect end support, $l_{b,Zwi}$: intermediate support

with hooks:	l_b = 80.7 cm,	$l_{b,min}$ = 20.0 cm,	$l_{b,net}$ = 55.4 cm
	$l_{b,dir}$ = 36.9 cm,	$l_{b,ind}$ = 55.4 cm,	$l_{b,Zwi}$ = 12.0 cm
without:	l_b = 80.7 cm,	$l_{b,min}$ = 24.2 cm,	$l_{b,net}$ = 79.2 cm
	$l_{b,dir}$ = 52.8 cm,	$l_{b,ind}$ = 79.2 cm,	$l_{b,Zwi}$ = 12.0 cm

reinforcement drawing:

scale 1 : 10

cvo = 2.5 cm
cvu = 3.5 cm



cross-section data

gross area of concrete: A_c = 8.0 dm^2, second moment of area: I_{cs} = 10.7 dm^4
moment of resistance: W_{cs} = 5.3 dm^3, distance of centre of gravity from upper edge: z_s = 20.0 cm
total area of longitudinal reinforcement: $\Sigma(\min A_s)$ = 17.6 cm^2 $\Rightarrow$ ρ_s = 2.20% < 8.00%

material properties for design calculation

concrete	f_{ck}	α	ε_{c2}	ε_{c2u}	n_c	E_{cm}	f_{ctm}
	MN/m^2	-	‰	‰	-	MN/m^2	MN/m^2
C25/30	25.0	0.850	-2.00	-3.50	2.00	31475.8	2.565

design value of compression strength $f_{cd} = \alpha_c f_{ck} / \gamma_c$
strain at reaching the maximum strength ε_{c2}, ult. compr. strain ε_{c2u}
concr. comp. stress $\sigma_c = f_{cd} (1-(1-\varepsilon_c/\varepsilon_{c2})^n)$ for $0 \geq \varepsilon_c > \varepsilon_{c2}$ and $\sigma_c = f_{cd}$ for $\varepsilon_{c2} \geq \varepsilon_c > \varepsilon_{c2u}$
modulus of elasticity E_{cm}, mean value of axial tensil strength f_{ctm}

reinforcem.	f_{yk}	f_{tk}	ε_{su}	E_s
	MN/m^2	MN/m^2	‰	MN/m^2
BSt 500 (A)	500.0	525.0	25.00	200000.0

design yield strength $f_{yd} = f_{yk} / \gamma_s$
design tensile strength $f_{td} = f_{tk} / \gamma_s$
ult. tensile strain ε_{su}, modulus of elasticity E_s

Applied design parameters of the national appendix

Germany

DIN EN 1992-1-1 (EC 2)

Chapter	Value	Meaning
2.4.2.4(1)		Partial safety factors for concrete and reinforcement
	γ_c = 1.50 γ_s = 1.15	Permanent and transient design situation
	γ_c = 1.50 γ_s = 1.15	Fatigue design situation
	γ_c = 1.50 γ_s = 1.15	Earthquake design situation
	γ_c = 1.30 γ_s = 1.00	Accidental design situation
3.1.6(1)P	α_{cc} = 0.85	Reduction factor of compression strength of concrete
3.1.6(2)P	α_{ct} = 1.00	Reduction factor of tensile strength of concrete
6.2.2(1)	$C_{Rd,c}$ = 0.15 / γ_c	Coeff. to calculate the resistance of shear force
	v_{min} = 0.0525/γ_c $k^{3/2}$ $f_{ck}^{1/2}$	
	k_1 = 0.12	
6.2.2(6)	v_V = 0.675	reduction factor of strength of shear force
6.3.2(4)	v_T = 0.525	reduction factor of strength of torsion
6.2.3(2)	min cot Θ = 1.00	lower limit of strut gradient
	max cot Θ = 3.00	upper limit of strut gradient
6.2.3(3)	α_{cw} = 1.00	Coeff. to consider the state of stress in the compress. boom
	v_1 = 0.750	Coeff. to calculate the max. design resistance of shear force
6.2.4(4)	cot Θ_{fz} = 1.00	Connections: strut gradient of tension booms
	cot Θ_{fd} = 1.20	Connections: strut gradient of compression booms
6.2.4(6)	k = 0.00	Connections: Coeff. of resisting tensile stress without through-reinforcement
6.2.5(2)	intended : c= 0.50, μ= 0.90	Joints: coefficients of roughness
	rough : c= 0.40, μ= 0.70	
	smooth : c= 0.20, μ= 0.60	
	very smooth: c= 0.00, μ= 0.50	
6.8.4(1)	$\gamma_{F,fat}$ = 1.00	Fatigue: Safety factor of action effects
6.8.7(1)	k_1 = 1.00	Fatigue: Coeff. to calculate the design strength of concrete
7.3.4(3)	k_3 = 0.00	Cracks: Coeff. to calculate the maximum crack distance if fracture pattern is completed
	k_4 = 0.278	Cracks: Coeff. to calculate the maximum crack distance if fracture pattern is completed
9.2.1.1(1)	$A_{s,min}$ s. NA-DE	minimum reinforcement of beams and slab n [cm²]
9.2.2(5)	$\rho_{w,min}$ s. NA-DE	minimum ratio of shear reinforcement
9.5.2(2)	$A_{s,min}$ = 0.150 N_{Ed} / f_{yd}	
	≥ 0.000 A_c	minimum reinforcement for columns [cm²]
9.6.2(1)	$A_{s,vmin}$ s. NA-DE	vertikal minimum reinforcement for walls [cm²]
11.3.5(1)	α_{lcc} = 0.75	Lightw.conc.: Reduction factor of compression strength of concrete
11.3.5(2)	α_{lct} = 1.00	Lightw.conc.: Reduction factor of tensile strength of concrete
11.6.1(1)	$C_{lRd,c}$ = 0.15 / γ_c	Lightw.conc.: Coeff. to calculate the resistance of shear force
	$v_{l,min}$ = 0.0525 $k^{3/2}$ $f_{lck}^{1/2}$	
	k_{l1} = 0.12	
11.6.1(2)	v_l = 0.675 η_1	Lightw.conc.: reduction factor of strength of shear force
	v_l = 0.525 η_1	Lightw.conc.: reduction factor of strength of torsion
11.6.2(1)	v_{l1} = 0.750 η_1	Lightw.conc.: Coeff. to calculate the maximum shear resistance
12.3.1(1)	$\alpha_{cc,pl}$ = 0.70	unreinf.concr.: Reduct. factor of compr. strength of concrete
	$\alpha_{ct,pl}$ = 0.70	unreinf.concr.: Reduct. factor of tens. strength of concrete
12.6.3(2)	k_p s. NA-DE	unreinf.concrete: coefficient for design stress analysis in shear design

DIN EN 1992-1-2 (EC 2, fire)

Chapter	Value	Meaning
3.2.3(5)	class N (table 3.2a)	reinforcement-class to describe stress-strain-relation at increased temperatures
3.3.3(1)	λ_c = λ_{co} oder λ_{cu}	thermal conductivity of concrete
	see design calc. options	λ_{co} upper limit, λ_{cu} lower limit acc. to 3.3.3(2)
6.1(5)	class 1 (table 6.1N)	high strength concrete: concrete-class to describe the reduction of strength
6.4.2.1(3)	k = 1.000	high strength concrete: coefficient for cross-section reduction
6.4.2.2(2)	k_m = 1.000	high strength concrete: Coeff. for moment load capacity

Chapter	Value	Meaning
		under fire load in the tension zone